Validation of VAMUCH for Fiber Reinforced Composites

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To show the predictive capability of VAMUCH for unidirectional fiber reinforced composites, we choose a few examples extensively studied in the literature. For the first two examples, comparisons are made between the finite element method (FEM) of Sun and Vaidya,[1] method of cell (MOC),[2] generalized method of cell (GMC),[3] high-fidelity generalized method of cell (HFGMC),[4] and elasticity-based cell method (ECM).[5] The FEM approach in Ref. 1 is established on a rigorous mechanics foundation and 3D representative volume elements with periodic boundary conditions are used for homogenization. The MOC and its variants (GMC, HFGMC, and ECM) expand the local displacements in terms of global displacements using the Legendre polynomials of different orders. ECM starts from this assumption and solve the equations of continuum mechanics in a strong form. In contrast, MOC, GMC, and HFGMC invokes additional *ad hoc* assumptions such as that the interfacial continuous conditions and periodic boundary conditions are only satisfied in the integral sense. FEM results are directly taken from Ref. 1, MOC and ECM results from Ref. 5, GMC and HFGMC results from Ref. 4.

Table 1. Effective material properties of boron/aluminum composites

Models	E_{11} (GPa)	E_{22} (GPa)	G_{12} (GPa)	G_{23} (GPa)	ν_{12}	ν_{23}
VAMUCH	215.3	144.1	54.39	45.92	0.195	0.255
FEM[1]	215	144	57.2	45.9	0.19	0.29
MOC[2]	215	142.6	51.3	43.7	0.20	0.25
GMC[3]	215.0	141.0	51.20	43.70	0.197	0.261
HFGMC[4]	215.4	144.0	54.34	45.83	0.195	0.255
ECM[5]	215	143.4	54.3	45.1	0.19	0.26

The first example is boron/aluminium composites. Both constituents are isotropic with Young's modulus $E = 379.3$ GPa and Poisson's ratio $\nu = 0.1$ for boron fibers, and $E = 68.3$ GPa and Poisson's ratio $\nu = 0.3$ for aluminium matrix. The fiber is of circular shape and arranged in a square array (see the sketch in the middle of Figure **??**) and the fiber volume fraction is 0.47. The effective material properties predicted by different approaches are listed in Table 1. It can be observed that MOC and GMC significantly underpredict the shear moduli G_{12} and G_{23} while FEM overpredicts the longitudinal shear modulus G_{12}. The closest correlation for all the values is found between VAMUCH and HFGMC.

The second example is graphite/epoxy composites. Graphite fiber is transversely isotropic with $E_{11} = 235$ GPa, $E_{22} = 14$ GPa, $G_{12} = 28$ GPa, $\nu_{12} = 0.2$, and $\nu_{23} = 0.25$. Epoxy matrix is isotropic with Young's modulus $E = 4.8$ GPa and Poisson's ratio $\nu = 0.34$. The fiber is circular and arranged in a square array

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and the fiber volume fraction is 0.6. The results from different approaches are listed in Table 2. Again the closest correlation is found between VAMUCH and HFGMC. Considering the fact that HFGMC uses the governing equations of MHT and that VAMUCH can reproduce MHT, it is not surprising to find out that HFGMC has an excellent agreement with VAMUCH. The FEM and ECM predictions are also very close to VAMUCH results for this case. It is noted that the ECM results listed in Tables 1 and 2 are obtained from 3rd order model. If the 5th order theory is used, the correlation between ECM and VAMUCH might be improved as shown in the following two examples.

Table 2. Effective material properties of graphite/epoxy composites

Models	E_{11} (GPa)	E_{22} (GPa)	G_{12} (GPa)	G_{23} (GPa)	ν_{12}	ν_{23}
VAMUCH	142.9	9.61	6.10	3.12	0.252	0.350
FEM[1]	142.6	9.60	6.00	3.10	0.25	0.35
MOC[2]	143	9.6	5.47	3.08	0.25	0.35
GMC[3]	143.0	9.47	5.68	3.03	0.253	0.358
HFGMC[4]	142.9	9.61	6.09	3.10	0.252	0.350
ECM[5]	143	9.6	5.85	3.07	0.25	0.35

In the following two examples, VAMUCH is compared with MOC, ECM (both 3rd order and 5th order), Green's function based approach (G-F),[6] and FEM. The results of MOC, ECM, and G-F are directly taken from Ref. 5, while FEM results are calculated using ANSYS following the approach proposed in Ref. 1. To be consistent with Ref. 5, we use 2D UCs having square inclusions in the center for VAMUCH. As pointed out in Ref. 5, square inclusions provide a stringent test of correct modeling the local and global behavior of heterogeneous materials due to strong gradients in the local fields induced by the corners. The two material systems we consider are tungsten/copper composite and void/copper composite. Both tungsten and copper are assumed to be isotropic with $E = 395.0$ GPa and Poisson's ratio $\nu = 0.28$ for tungsten, and $E = 127.0$ GPa and Poisson's ratio $\nu = 0.34$ for copper.

Table 3. E_{22} (GPa) of W/Cu composites varying with fiber volume fraction

Models	0.0204	0.1837	0.5102	0.7511
VAMUCH	129.92	156.51	229.72	300.99
FEM[1]	129.92	156.51	229.71	301.0
G-F[6]	129.87	156.18	229.09	300.70
MOC[2]	129.50	154.40	226.20	299.00
ECM (3rd order)[5]	129.50	154.60	226.60	299.10
ECM (5th order)[5]	129.80	156.50	229.50	300.80
VAMUCH (circular)	129.81	155.19	226.94	298.12
FEM[1] (circular)	129.82	155.20	226.97	298.14

For both material systems, we calculate the effective transverse Young's modulus at different inclusion volume fractions of 0.0204, 0.1837, 0.5102, and 0.7511. The results are listed in Table 3 for tungsten/copper composite and Table 4 for void/copper composite. It is verified that the FEM approach of Sun and Vaidya has no size effects for E_{22}, which means this approach will provide the most accurate prediction of E_{22} with a converged mesh. As one can observe from Tables 3 and 4, MOC and 3rd order ECM underpredict this value up to 1.6% for tungsten/copper composite and 8.6% for void/copper composite. VAMUCH, G-F, and 5th order ECM have excellent agreement with FEM, with VAMUCH having the closest correlations.

Table 4. E_{22} (GPa) of Void/Cu composites varying with void volume fraction

Models	0.0204	0.1837	0.5102	0.7511
VAMUCH	120.22	81.73	39.75	18.25
FEM[1]	120.22	81.70	39.75	18.25
G-F[6]	120.63	83.50	40.48	18.40
MOC[2]	110.20	75.27	38.22	17.99
ECM (3rd)[5]	110.20	75.38	38.23	17.99
ECM (5th)[5]	118.90	80.97	39.64	18.20
VAMUCH (circular)	120.34	82.67	39.08	10.31
FEM[1] (circular)	120.34	82.64	39.08	10.31

To show the effect of shape of inclusions, we predict the effective transverse Young's modulus using UC with circular inclusions arranged in a square array. As shown in Tables 3 and 4, the shape effects of inclusions become more and more significant and cannot be neglected with large volume fraction of inclusions, particularly for voided composites. For example, E_{22} of void/composite with square holes is 80% larger than the composite with circular holes, although the void volume fraction remains the same. It is interesting to note that E_{22} of the W/Cu composite with square inclusions is slightly larger than that with circular inclusions, with the difference getting bigger with larger fiber volume fraction. However, for the case of Void/Cu, material with square voids is slightly smaller than that with circular voids. As the void volume fraction getting bigger the trend is reversed. A parametric study is carried out to find out the point where the trend is reversed, the results of which are plotted Figure 1. As observed from this figure, when the void volume fraction is greater than zero and less than 0.45 (approximate), E_{22} of materials having square voids are slightly smaller than those having circular voids. When the void volume fraction is greater than 0.45 (approximate), materials with square voids have bigger E_{22} that those with circular voids.

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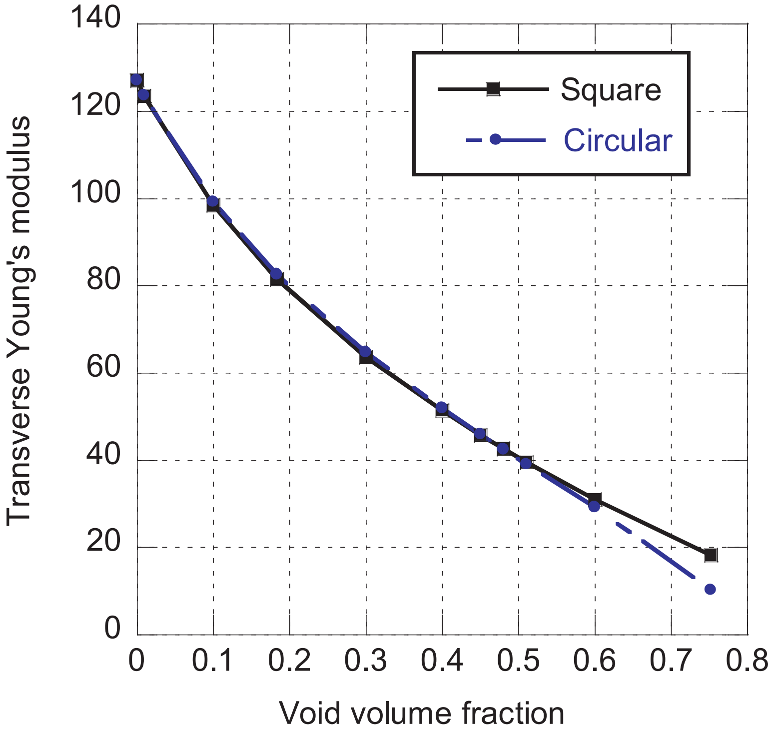

Figure 1. Change of Young's modulus of material with square voids and circular voids with respect to void volume fractions