Thermoelastic Micromechanical Analysis Using VAMUCH

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In this paper, we will use some examples to demonstrate VAMUCH capabilities for predicting thermoelastic properties and local stress field due to temperature changes.

I. Predict effective CTEs

Exact solutions of effective CTEs exist for isotropic or transversely isotropic composites having isotropic constituents[1] if the effective elastic properties are know. Sincere there are no exact formulas existing for general UCs and VAMUCH achieves the best accuracy for predicting effective elastic properties,[2] we will consider the elastic properties predicted by VAMUCH as exact and use them to calculate the exact solutions of effective CTEs following the closed-form solution given in Ref. 1. Such solutions can serve as excellent benchmarks for VAMUCH thermoelastic capabilities.

The first example is boron/aluminium composites. Both constituents are isotropic with Young's modulus $E = 379.3$ GPa, Poisson's ratio $\nu = 0.1$, and CTE $\alpha = 8.1\ 10^{-6}/°\text{C}$ for boron fibers, and $E = 68.3$ GPa, Poisson's ratio $\nu = 0.3$ for aluminium matrix, and CTE $\alpha = 23.0\ 10^{-6}/°\text{C}$. The fiber is of circular shape and arranged in a square array. The example of fiber volume fraction 0.47 is studied in several places.[3–5] The effective CTEs predicted by different approaches are listed in Table 1. It can be observed that VAMUCH has a perfect match with exact solution up to the fourth significant digits. HFGMC also has an excellent agreement with the exact solution while GMC and Tamma and Avila's results are not so accurate. To show the trend of change of effective CTEs with respect to the change of fiber volume fraction, we plot the effective CTEs for the same composites with different fiber volume fractions in Figure 1 and Figure 2. As it is expected, both axial CTEs and transverse CTEs are decreasing with increasing fiber volume fractions. Again the perfect match between exact solutions and VAMUCH is noticed.

Table 1. Effective CTEs of boron/aluminum composites

Models	$\alpha_{11}(10^{-6}/°\text{C})$	$\alpha_{22}(10^{-6}/°\text{C})$
Exact Solution[1]	10.99	16.69
VAMUCH	10.99	16.69
HFGMC[3]	11.00	16.70
GMC[5]	10.91	16.94
Tamma and Avila[4]	10.77	17.34

The second example is to predict the effective Young's modulus for a glass/epoxy composite. The UC of this composite is composed of glass spheres embedded in a triply periodic cubic array. Both constituents

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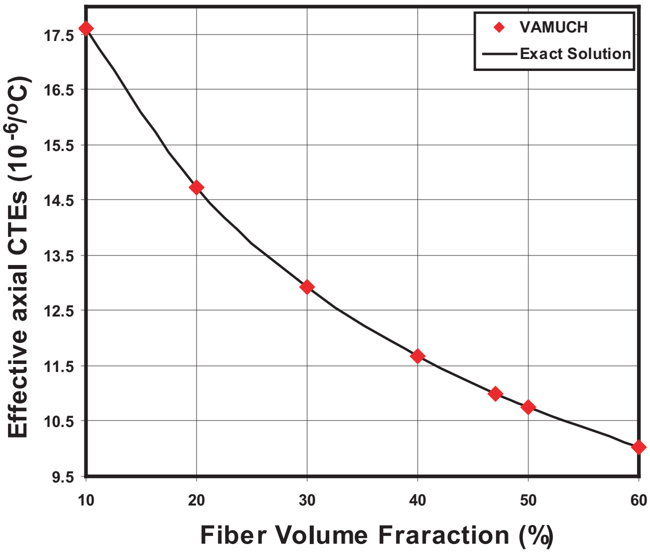

Figure 1. Effective α_{11} change with respect to the fiber volume fraction

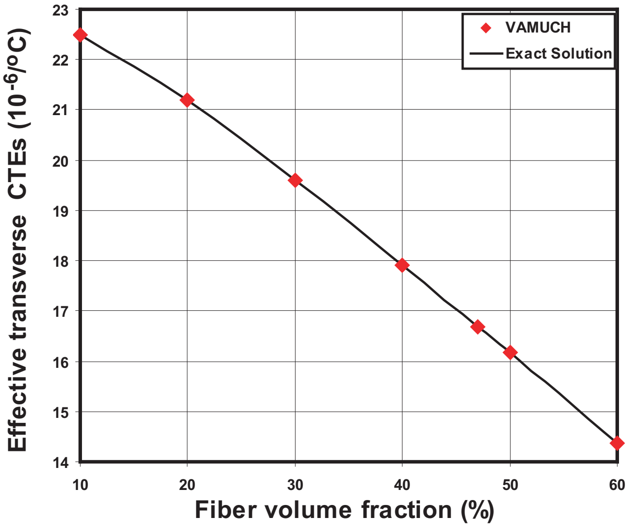

Figure 2. Effective α_{22} change with respect to the fiber volume fraction

are isotropic with Young's modulus $E = 76.00$ GPa, Poisson's ratio $\nu = 0.23$, and CTE $\alpha = 5.0\ 10^{-6}/°\mathrm{C}$ for glass, and Young's modulus $E = 3.01$ GPa, Poisson's ratio $\nu = 0.394$, and CTE $\alpha = 54.0\ 10^{-6}/°\mathrm{C}$ for epoxy. We plot the change of effective CTE with respect to the particle volume fraction in Figure 3. Again, it is found out that VAMUCH results are right on the top of exact solutions, which demonstrates VAMUCH provides accurate predictions for CTEs for particle reinforced composites.

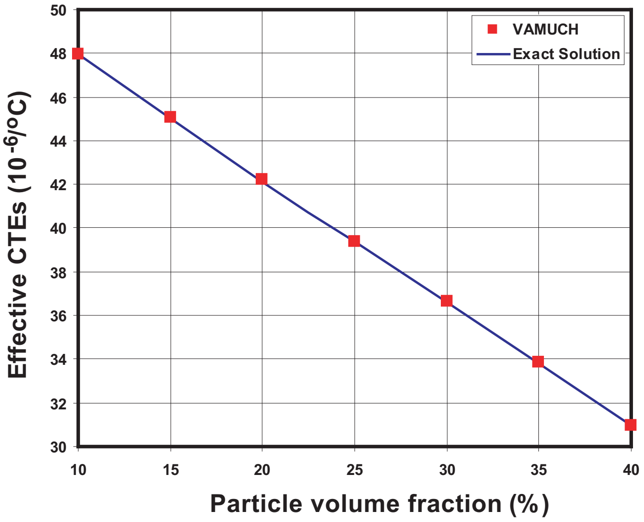

Figure 3. Effective CTE of glass/epoxy composite with spherical inclusions

II. Predict local thermal stresses

Finally, we can use VAMUCH to recover the stress distribution within the UC due to macroscopic temperature change. Consider the boron/aluminum composite with fiber volume fraction as 0.2. We use the effective thermoelastic properties to carry out a macroscopic thermoelastic analysis of the homogenized material. Suppose we know for a certain UC, it is stress free, yet the temperature is increased by 100°C. Due to the mismatch of CTEs of the constituents, thermal stresses will be generated within the UC. The distributions of σ_{22} and σ_{23} are plotted in Figures 4 and 5, respectively. All the sudden changes of stress distributions along the fiber-matrix interface have been well captured by VAMUCH. Such a capability of VAMUCH is very useful for analysis of residual stresses caused by temperature changes during manufacturing or operating process.

References

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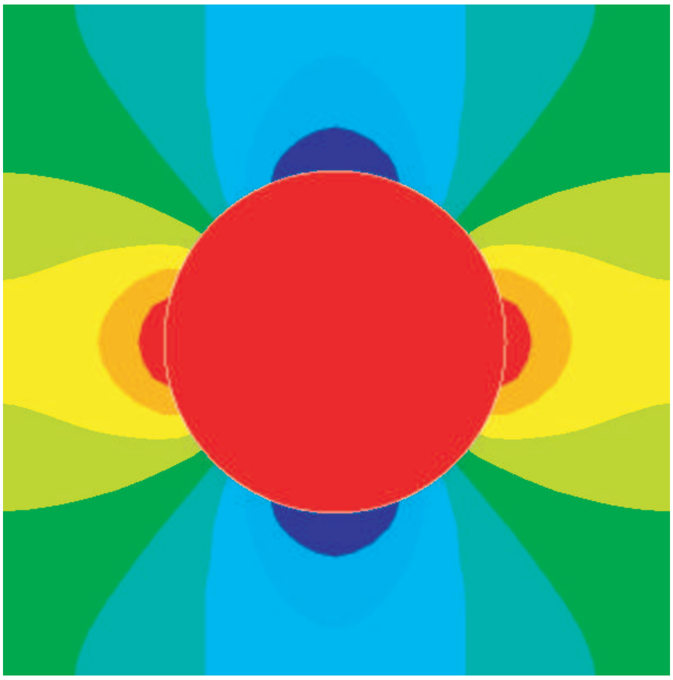

Figure 4. Contour plot of σ_{22} within the UC due to temperature change

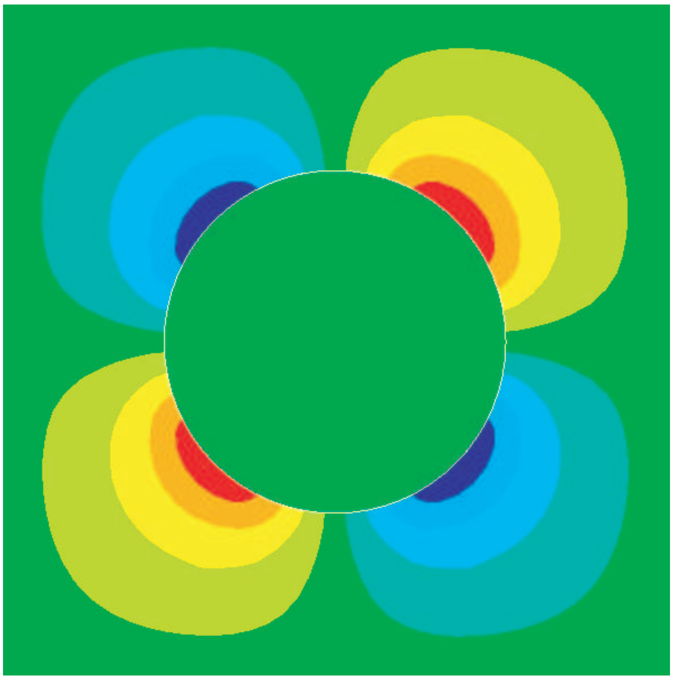

Figure 5. Contour plot of σ_{23} within the UC due to temperature change

Materials, **14**, pp. 127–139.